Negative refraction in (bi)-isotropic periodic arrangements of chiral SRRs

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(Dated: February 2, 2008)

Bi-isotropic and isotropic negative refractive index (NRI) 3D metamaterials made from periodic arrangements of chiral split ring resonators (SRRs) are proposed and demonstrated. An analytical theory for the characterization and design of these metamaterials is provided and validated by careful full-wave electromagnetic simulations. The reported results are expected to pave the way to the design of practical 3D bi-isotropic and isotropic NRI metamaterials made from a single kind of inclusions.

PACS numbers: 41.20.Jb, 42.70.Qs, 78.20.Ci, 78.20.Ek

Most proposals for developing negative refractive index (NRI) metamaterials (e.g. [1], [2], [3]) make use of two kind of inclusions, one of them to provide negative permittivity and the other to provide negative permeability. On the other hand, artificial media made from metallic chiral inclusions have been known since far by physicists [4], [5]. It is known that they present the most general linear and reciprocal constitutive relations [6] which, assuming 3D spatial isotropy and a time dependence of the kind $\exp(-j\omega t)$, can be written as

$$\mathbf{D} = \varepsilon_0(1+\chi_e)\mathbf{E} + j\sqrt{\varepsilon_0\mu_0}\,\kappa\mathbf{H} \quad (1)$$

$$\mathbf{B} = -j\sqrt{\varepsilon_0\mu_0}\,\kappa\mathbf{E} + \mu_0(1+\chi_m)\mathbf{H}\,, \quad (2)$$

where χ_e, χ_m and κ are the electric, magnetic and cross-polarizabilities of the metamaterial which, for lossless media, are real quantities. Media satisfying (1) and (2) are usually called *bi-isotropc*, to differentiate from usual isotropic media with $\kappa = 0$. Since they present simultaneous electric and magnetic properties, it has been guessed that they may open the way to NRI metamaterials made of a single kind of inclusions. To the best of our knowledge, the first proposal in this direction was made by S.Tretyakov in [7], and further developed in [8]. Also, in [9] a mixture of metallic chiral inclusions and wires was proposed in order to obtain negative refraction.

Eigenwaves propagating through reciprocal bi-isotropic media are right- and left-handed circulary polarized plane waves, whose dispersion relation is given by [6]

$$k^{\pm} = k_0\left(\sqrt{\mu_r\varepsilon_r} \pm \kappa\right)\,, \quad (3)$$

where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, and $\varepsilon_r = (1+\chi_e)$, $\mu_r = (1+\chi_m)$. In order to reduce forbidden bands of propagation coming from complex values of $k^{\pm}$, it is desirable that $\chi_e(\omega) \simeq \chi_m(\omega)$ so that μ_r and ϵ_r always have the same sign. As explained in [8] and [10], usually this condition also implies that

$$\chi_e(\omega) \simeq \chi_m(\omega) \simeq |\kappa(\omega)|\,. \quad (4)$$

The conditions for negative refraction in transparent bi-isotropic chiral media were investigated in [11]. In lossless bi-isotropic media satisfying the *balance* condition (4), these conditions reduces to $\varepsilon_r < 0.5$ and $\mu_r < 0.5$ [10], which is less restrictive than the usual condition, $\varepsilon_r < 0$ and $\mu_r < 0$, for conventional isotropic medium (i.e., a medium with $\kappa = 0$). The price to pay for this enhanced bandwidth is that only one of the eigenwaves of (3) exhibits negative refraction, the other presenting positive refraction [10].

Most previously proposed and studied bi-isotropic artificial media (see, for instance, [8], [10] and references therein) were *random* arrangements of chiral inclusions. Random arrangements have the advantage of ensuring isotropy provided the number of inclusions per unit volume is high enough. However, multiple scattering in random arrangements of inclusions affects the effective constitutive parameters of the mixture, leading to an imaginary part of the susceptibilities to account for radiation losses [6]. This important drawback of radiation losses can be avoided by using *periodic* arrangements of chiral scatterers. Periodic arrangements also provide higher density of scatterers, which will result in stronger electromagnetic responses and wider NRI frequency bands. In addition, periodic arrangements can provide much better realiability and reproducibility than random mixtures. Finally, wave propagation through periodic structures can be studied by means of standard electromagnetic solvers, thus providing a validation method for the homogenization procedure that has no counterpart in random structures. For all these reasons periodic arrangements of chiral inclusions are preferred over random mixtures. However, care must be taken to obtain a truly isotropic periodic metamaterial in order to keep the appropriate symmetries of the crystal; otherwise couplings between elements may destroy isotropy even if the structure appears as isotropic in the dipolar approximation [12]. In [12] it has been shown that a cubic arrangement of SRRs satisfying the thetraedron or (in Schoenflies notation) T-group of symmetry provides an useful basic design for this purpose. Fig.1.a shows a cubic arrangement of chiral SRRs satisfying this symmetry. It is a combi-

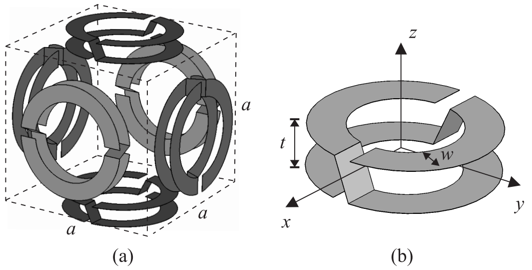

FIG. 1: (a) Cubic arrangement of chiral SRRs satisfying T-group of symmetry (namely, any of the inbody diagonals is a third order rotation symmetry axis), and (b) detail of the chiral SRR design.

nation of six identical chiral SRRs, which are shown in detail in Fig.1.b. The proposed chiral SRR is a modification of the original SRR design [13] that substitutes the edge-coupling between rings by a broad-side coupling [14], also introducing some helicity. It should be noticed that the SRRs on opposite sides of the cube of Fig.1.a are identical, which makes it possible to develop a periodic metamaterial with a single cubic lattice from this design. The unit cell of this metamaterial has periodicity a (the edge of the cube) and will contain three chiral SRRs. Note that the proposed chiral-SRRs is expected to be a feasible "practical" design because of its possible manufacture via standard planar-circuit fabrication techniques[18].

The chiral SRR polarizabilities are defined by

$$\mathbf{p} = \overline{\overline{\alpha}}^{\rm ee} \cdot \mathbf{E}_{\rm l} + \overline{\overline{\alpha}}^{\rm em} \cdot \mathbf{B}_{\rm l} \tag{5}$$

$$\mathbf{m} = -(\overline{\overline{\alpha}}^{\rm em})^{\rm T} \cdot \mathbf{E}_{\rm l} + \overline{\overline{\alpha}}^{\rm mm} \cdot \mathbf{B}_{\rm l}\,, \tag{6}$$

where $(\cdot)^{\rm T}$ means transpose, $\mathbf{p}$ and $\mathbf{m}$ are the electric and magnetic dipoles induced in the chiral SRR, $\mathbf{E}_{\rm l}$ and $\mathbf{B}_{\rm l}$ are the electric and magnetic local fields *seen* by the SRR, and Onsager symmetries [15] for the magneto-electric susceptibilities $\overline{\overline{\alpha}}^{\rm me} = -(\overline{\overline{\alpha}}^{\rm em})^{\rm T}$ have been explicitly introduced. Analytical expressions for these polarizabilities can be obtained by a straighforward extension of the theory reported in [14]. The results are [16]

$$\alpha_{zz}^{\rm mm} = \alpha_0^{\rm mm} X(\omega)\,; \qquad \alpha_0^{\rm mm} = \frac{\pi^2 r^4}{L} \tag{7}$$

$$\alpha_{zz}^{\rm em} = = \pm\alpha_0^{\rm em}\left(\frac{\omega_0}{\omega}\right) X(\omega); \quad \alpha_0^{\rm em} = j\frac{2\pi r^2 t}{\omega_0 L} \tag{8}$$

$$\alpha_{zz}^{\rm ee} = \alpha_0^{\rm ee}\left(\frac{\omega_0}{\omega}\right)^2 X(\omega)\,; \qquad \alpha_0^{\rm ee} = \frac{(2t)^2}{\omega_0^2 L} \tag{9}$$

$$\alpha_{xx}^{\rm ee} = \alpha_{yy}^{\rm ee} = \alpha_0\,; \qquad \alpha_0 = \varepsilon_0 \frac{16}{3} r_{\rm ext} \tag{10}$$

where $r_{\rm ext}$ is the external radius of the inclusion, $r = r_{\rm ext} - w/2$ is the average radius, and $X(\omega)$ is a common resonant factor given by

$$X(\omega) = \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L}\,. \tag{11}$$

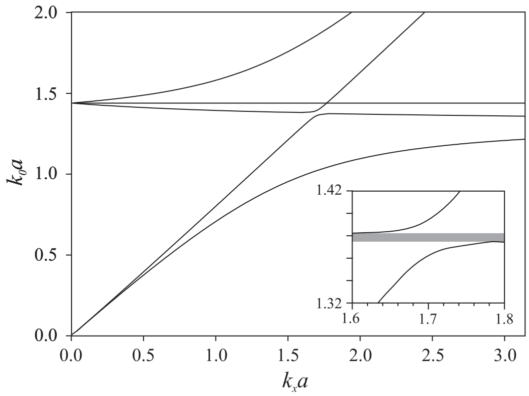

FIG. 2: Theoretical dispersion curves for the eigenwaves of the periodic metamaterial made of the periodic repetition of the cubic arrangement of Fig.1.a, forming a single cubic (sc) lattice with peridocity a. SRR parameters are $a/r_{\rm ext} = 2.87$, $w/r_{\rm ext} = 0.5$, and $t/r_{\rm ext} = 0.47$. Dispersion curves are drawn inside the first Brillouin zone of the structure.

The frequency of resonance $\omega_0 = 1/\sqrt{LC}$ of the inclusion, as well as the self inductance, L, the capacitance, C, and the resitance, R, coincide with those of the aforementioned broadside-coupled SRR [14]. From (7)–(10) the average magnetic-, electric- and cross-polarizability per particle in the isotropic metamaterial are easily obtained as $\langle\alpha_{\rm m}\rangle = \alpha_{zz}^{\rm mm}/3$, $\langle\alpha_{\rm e}\rangle = \alpha_{zz}^{\rm ee}/3 + \alpha_{xx}^{\rm ee}/3 + \alpha_{yy}^{\rm ee}/3$, and $\langle\alpha_{\rm em}\rangle = \alpha_{zz}^{\rm em}/3$ respectively. From these expresions, the susceptibilities of the metamaterial can be obtained from a straighforward extension of Lorentz local field theory that takes into account the presence of cross-polarizabilities. This theory leads to the following equations for the macroscopic polarization, $\mathbf{P}$, and magnetization, $\mathbf{M}$, of the metamaterial:

$$\mathbf{P} = N\left\{\langle\alpha_{\rm e}\rangle\left(\mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}\right) + \mu_0\langle\alpha_{\rm em}\rangle\left(\mathbf{H} + \frac{\mathbf{M}}{3}\right)\right\} \tag{12}$$

$$\mathbf{M} = N\left\{\mu_0\langle\alpha_{\rm m}\rangle\left(\mathbf{H} + \frac{\mathbf{M}}{3}\right) - \langle\alpha_{\rm em}\rangle\left(\mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}\right)\right\} \tag{13}$$

where $N = 3/a^3$ is the number of particles per unit volume, and $\mathbf{E}$, $\mathbf{H}$ are the macroscopic fields. Using (12)–(13) the metamaterial susceptibilities are directly obtained from their definitions $\chi_e = \mathbf{E}/(\varepsilon_0\mathbf{P})$ and $\chi_m = \mathbf{H}/\mathbf{M}$. The balance condition (4) is translated to the chiral-SRR polarizabilities as $\langle\alpha_{\rm m}\rangle = |\langle\alpha_{\rm em}\rangle|/c = \langle\alpha_{\rm e}\rangle/c^2$ where c is the velocity of light. If the effect of the non-resonant polarizability (10) is neglected, the above condition reduces to the following approximate balance condition:

$$t\lambda_0 = (\pi r)^2\,, \tag{14}$$

where λ_0 is the wavelength at resonance.

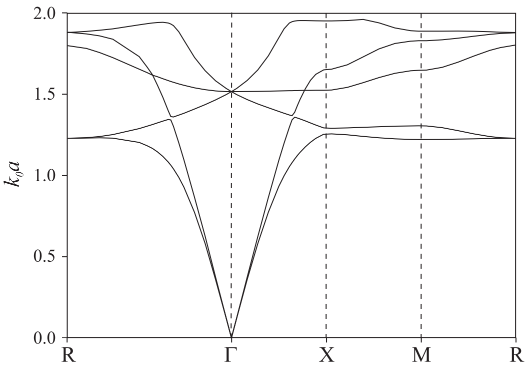

FIG. 3: Band structure and dispersion relation for the metamaterial of Fig.2 obtained from full-wave simulations along the path R-Γ-X-M-R.

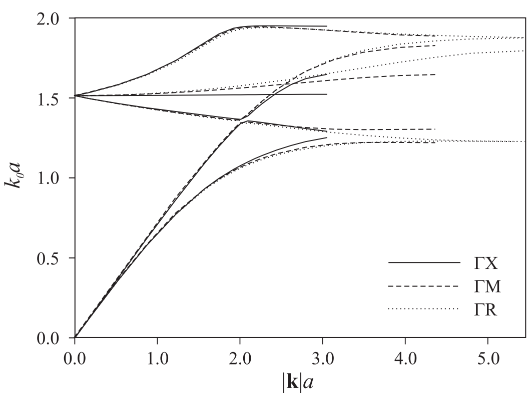

FIG. 4: Dispersion curves along the Γ-X, Γ-M, and Γ-R directions for the eigenwaves of the balanced metamaterial of Figs. 2 and 3.

In the following a particular example is analyzed in order to illustrate and validate our analytical theory. The chosen metamaterial parameters satisfy Eq. 14 and are given in the caption of Fig.2. This figure shows the theoretical dispersion curves for the eigenwaves of the bi-isotropic metamaterial. A region of backward-wave propagation for one of the eigenwaves can be clearly observed. This frequency band corresponds to the aforementioned condition $\varepsilon_r, \mu_r < 0.5$. Inside this band there is also a small forbidden band gap (see the gray region in the inset of the figure) that corresponds to complex values of the propagation constants (3). This small band gap is due to the aforementioned approximations implicit in Eq. 14. The straight horizontal line at the frequency where the propagation constant of one of the eigenwaves vanishes (corresponding to the condition $\varepsilon_r\mu_r = \kappa^2$) represents a fully degenerate longitudinal wave with $\mathbf{k} \times \mathbf{E} = 0$ and $\mathbf{k} \times \mathbf{H} = 0$.

The same structure has been simulated by using *CST Microwave Studio* and its band structure is shown in Fig.3. A good qualitative agreement is found between Figs. 2 and 3. In both figures a frequency band of

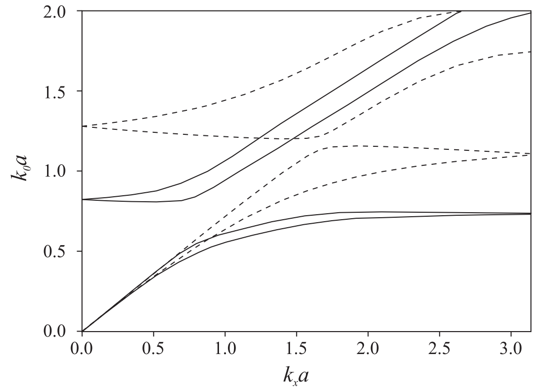

FIG. 5: Dispersion curves along the Γ-X direction for the transverse eigenwaves of two unbalanced metamaterials. Parameters are tha same as in Figs.2 to 4, except for t/r_{ext}, which is taken as $t/r_{\mathrm{ext}} = 0.3$ (dashed lines) and $t/r_{\mathrm{ext}} = 0.1$ (solid lines)

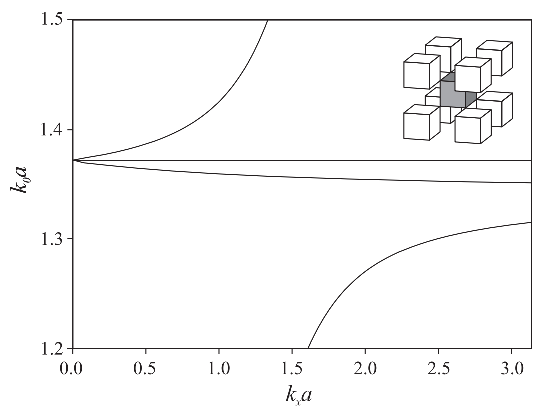

FIG. 6: Theoretical dispersion relation for plane waves propagating in the racemic metamaterial with the single cubic lattice shown in the inset. Inset: Racemic periodic structure made of two interleaved single cubic lattices of cubes as that of Fig.1.a. White cubes are identical to that of Fig.1.a, and gray cubes are similar cubes made of chiral SRRs of opposite handedness. Chiral SRR parameters are as in Fig.2 and periodicity of the structure is $2a$. The fully degenerate longitudinal mode is now located at $\varepsilon_r\mu_r = 0$

backward-wave propagation is observed for one of the eigenwaves. Also a small frequency stopband appears in Fig.3 inside this backward-wave passband (because of the approximate balance condititiion employed) in agreement with the theoretical predictions of Fig.2. In order to show the isotropy of the structure, the dispersion curves along the Γ-X, Γ-M and Γ-R contours are shown in Fig.4. The isotropy of the dispersion relation for the transverse modes becomes apparent from these curves, except for high values of the propagation constant, where spatial dispersion should affect the dispersion relation. The longitudinal mode of Fig.2 also appears in Figs.3 and 4, although now spatial dispersion destroys degeneracy and isotropy. It is illustrative to compare the band struc-

ture of the reported balanced structure with those of an unbalanced one. Thus, Fig.5 shows the dispersion diagrams for the transverse modes of two unbalanced structres (which were obtained by varying t in Fig.1). It can be observed how, as t decreases, the forbidden frequency band increases until it occupies all the backward-wave region, giving a behavior similar to that of a negative-μ split ring metamaterial [17]. This is expected, since the electric and magneto-electric polarizabilities of the chiral-SRR (8)-(9) decrease with t, until the chiral-SRR becomes an almost purely magnetic inclusion. Fig.5 illustrates the tolerance of the proposed design to deviations from the balance condition (14). As it can be seen in the figure (dashed lines) this tolerance is quite good.

Let us now explore the possibility of designing a racemic mixture of chiral SRRs while keeping the necessary symmetries to ensure an isotropic behavior for the metamaterial. For this purpose let us consider a single cubic lattice of peridicity $2a$ made of cubes as that shown in Fig.1.a, and another similar cubic lattice made of the same cubes but of opposite handedness Both cubic lattices can be interleaved as shown in the inset of Fig.6 to give a single cubic lattice whose unit cell is formed by two cubes of opposite handedness. Since the whole structure is symmetric after inversion (the centers of symmetry are the corners of the cubes), the cross suceptibility κ (which is a pseudo-scalar) must vanish [15]. Therefore the whole structure is a balanced isotropic metamaterial with $\kappa = 0$. Fig.6 shows the theoretical dispersion relation ($k = k_0\sqrt{\varepsilon_r \mu_r}$) of the proposed structure. A frequency band of backward-wave propagation, corresponding to the condition $\varepsilon_r, \mu_r < 0$, can be observed in the figure.

When designing a NRI metamaterial it is not sufficient to obtain some amount of negative refraction but it is also required to have small reflectance so that the refracted beam has a significant amplitude. As it is well known, in isotropic media the reflectance is governed by the wave impedance $Z = Z_0\sqrt{\mu_r/\varepsilon_r}$, where $Z_0 = \sqrt{\mu_0/\varepsilon_0} \approx 377\,\Omega$ is the wave impedance of free space. For bi-isotropic media the same expression $Z = Z_0\sqrt{\mu_r/\varepsilon_r}$ holds [6]. Since for balanced metamaterials it is imposed that $\chi_e \simeq \chi_m$ in the frequency range of interest, it directly follows that $Z(\omega) \simeq Z_0(\omega)$, which ensures good matching to free space. This last feature is an additional relevant advantage of the proposed *balanced* bi-isotropic and isotropic NRI metamaterials.

In summary, periodic NRI metamaterials based on chiral SRRs have been proposed and demonstrated. An analytical theory has been developed for the characterization of such metamaterias, which has been validated by careful full wave electromagnetic simulations. It has been shown that single cubic lattices of chiral-SRRs can provide bi-isotropic NRI metamaterials with a well defined frequency band of backward-wave propagation for one of its plane-wave eigenstates. Besides, a racemic single cubic lattice of chiral SRRs satisfying the apropriate symmetry (T_h group in Schoenflies notation) has been proposed in order to provide NRI isotropic metamaterials whith a well defined band of backward wave propagation for all its plane-wave eigenstates. In both cases good matching to free space is expected at all frequencies.

This work has been supported by the Spanish Ministerio de Educación y Ciencia under projects TEC2007-68013-C02-01/TCM and TEC2007-65376/TCM, and by Spanish Junta de Andalucía under projects P06-TIC-01368 and TIC-253.

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